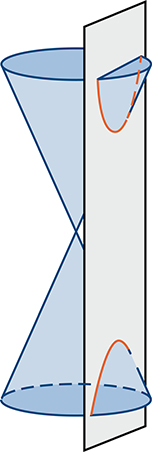
# Locating the Vertices and Foci of a Hyperbola

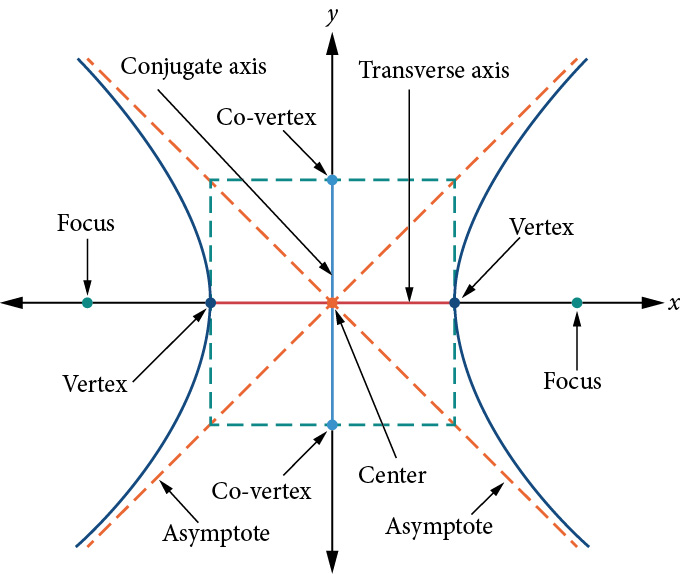
In analytic geometry, a **hyperbola** is a conic section formed by intersecting a right circular cone with a plane at an angle such that both halves of the cone are intersected. This intersection produces two separate unbounded curves that are mirror images of each other.



A hyperbola is defined as the set of all points in a plane such that the difference of the distances between and the foci is a positive constant.

Note that the hyperbola is defined in terms of the difference of two distances whereas the ellipse is defined in terms of the sum of two distances.

As with the ellipse, every hyperbola has two axes of symmetry. The **transverse axis** is a line segment that passes through the center of the hyperbola and has vertices as its endpoints. The foci lie on the line that contains the transverse axis. The **conjugate axis** is perpendicular to the transverse axis and has the co-vertices as its endpoints. The **center of the hyperbola** is the midpoint of both the transverse and conjugate axes, where they intersect. Every hyperbola also has two asymptotes that pass through its center. As a hyperbola recedes from the center, its branches approach these asymptotes. The **central rectangle** of the hyperbola is center at the origin with sides that pass through each vertex and co-vertex; it is a useful tool for graphing the hyperbola and its asymptotes. To sketch the asymptotes of the hyperbola, simply sketch and extend the diagonals of the central rectangle.



**The standard form of the equation of a hyperbola with center and transverse axis on the axis** is

where

• The length of the transverse axis is

• The coordinates of the vertices are

• The length of the conjugate axis is

• The coordinates of the co-vertices are

• The distance between the foci is , where

• The coordinates of the foci are

• The equations of the asymptotes are (see first figure)

**The standard form of the equation of a hyperbola with center and transverse axis on the axis** is

where

• The length of the transverse axis is

• The coordinates of the vertices are

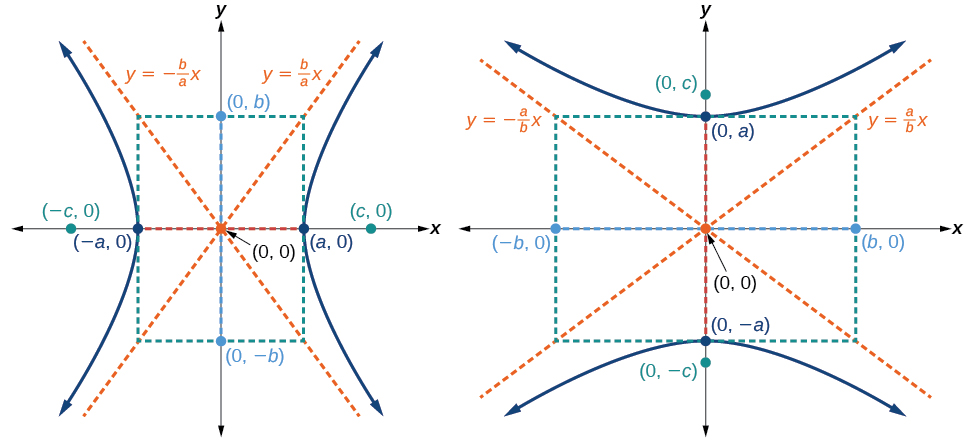
• The length of the conjugate axis is

• The coordinates of the co-vertices are

• The distance between the foci is , where

• The coordinates of the foci are

• The equations of the asymptotes are (see second figure)



Note that the vertices, co-vertices, and foci are related by the equation . When we are given the equation of a hyperbola, we can use this relationship to identify its vertices and foci.

**Given the equation of a hyperbola in standard form, locate its vertices and foci.**

1) Determine whether the transverse axis lies on the or axis. Notice that is always under the variable with the positive coefficient. So, if you set the other variable equal to zero, you can easily find the intercepts. In the case where the hyperbola is centered at the origin, the intercepts coincide with the vertices.

a. If the equation has the form , then the transverse axis lies on the axis. The vertices are located at , and the foci are located at .

b. If the equation has the form , then the transverse axis lies on the axis. The vertices are located at , and the foci are located at .

2) Solve for using the equation

3) Solve for using the equation

Examples

1. For the each of the following, determine whether the given equations represent hyperbolas. If yes, write in standard form.
2. Identify the vertices and foci of the hyperbola with equation .

# Writing Equations of Hyperbolas in Standard Form

Just as with ellipses, writing the equation for a hyperbola in standard form allows us to calculate the key features: its center, vertices, co-vertices, foci, asymptotes, and the lengths and positions of the transverse and conjugate axes.

## Hyperbolas Centered at the Origin

**Given the vertices and foci of a hyperbola centered at , write its equation in standard form.**

1) Determine whether the transverse axis lies on the - or -axis.

a. If the given coordinates of the vertices and foci have the form and , respectively, then the transverse axis is the -axis. Use the standard form .

b. If the given coordinates of the vertices and foci have the form and , respectively, then the transverse axis is the -axis. Use the standard form .

2) Find using the equation .

3) Substitute the values for and into the standard from of the equation determined in Step 1.

Example

What is the standard form equation of the hyperbola that has vertices and foci ?

## Hyperbolas Not Centered at the Origin

Like the graphs for other equations, the graph of a hyperbola can be translated. If a hyperbola is translated units horizontally and units vertically, the center of the hyperbola will be .

**The standard form of the equation of a hyperbola with center and transverse axis parallel to the -axis** is

where

• The length of the transverse axis is

• The coordinates of the vertices are

• The length of the conjugate axis is

• The coordinates of the co-vertices are

• The distance between the foci is , where

• The coordinates of the foci are

The asymptotes of the hyperbola coincide with the diagonals of the central rectangle. The length of the rectangle is and its width is . The slopes of the diagonals are , and each diagonal passes through the center . Using the **point-slope formula**, it is simple to show that the equations of the asymptotes are (see first figure).

**The standard form of the equation of a hyperbola with center and transverse axis parallel to the -axis** is

where

• The length of the transverse axis is

• The coordinates of the vertices are

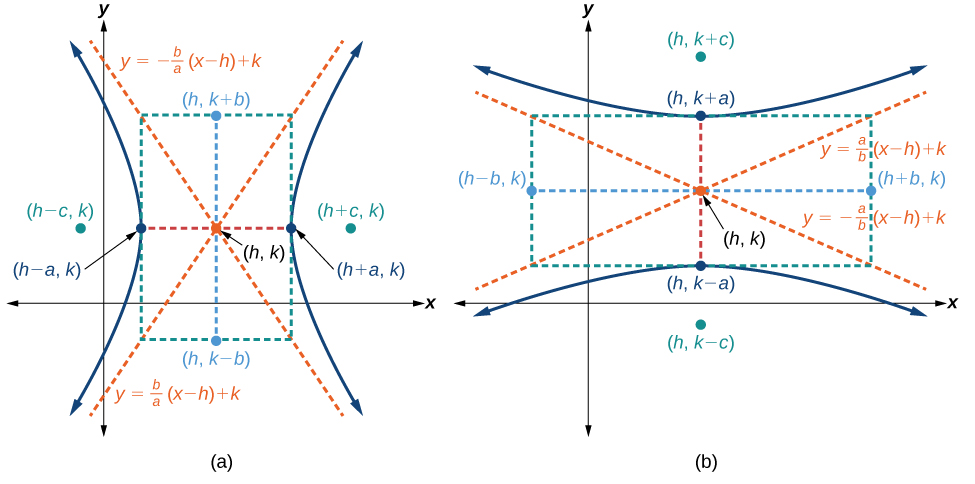
• The length of the conjugate axis is

• The coordinates of the co-vertices are

• The distance between the foci is , where

• The coordinates of the foci are

Using the reasoning above, the equations of the asymptotes are (see second figure).



**Given the vertices and foci of a hyperbola centered at , write its equation in standard form.**

1) Determine whether the transverse axis is parallel to the - or -axis.

a. If the -coordinates of the given vertices and foci are the same, then the transverse axis is parallel to the -axis. Use the standard form .

b. If the -coordinates of the given vertices and foci are the same, then the transverse axis is parallel to the -axis. Use the standard form .

2) Identify the center of the hyperbola, , using the midpoint formula and the given coordinates for the vertices.

3) Find by solving for the length of the transverse axis, , which is the distance between the given vertices.

4) Find using and found in Step 2 along with the given coordinates for the foci.

5) Solve for using the equation .

6) Substitute the values for and into the standard form of the equation determined in Step 1.

Example

What is the standard form equation of the hyperbola that has vertices at and and foci at and ?

# Graphing Hyperbolas

**Given a general form for a hyperbola centered at , sketch the graph.**

\*Note: hyperbolas centered at the origin have

1) Convert the general form to the standard form. Determine which of the standard forms applies to the given equation.

2) Use the standard form identified in Step 1 to determine the position of the transverse axis; coordinates for the center, vertices, co-vertices, and foci; and the equations for the asymptotes.

a. If the equation is in the form , then

• The transverse axis is parallel to the -axis

• The center is

• The coordinates of the vertices are

• The coordinates of the co-vertices are

• The coordinates of the foci are

• The equations of the asymptotes are

b. If the equation is in the form , then

• The transverse axis is parallel to the -axis

• The center is

• The coordinates of the vertices are

• The coordinates of the co-vertices are

• The coordinates of the foci are

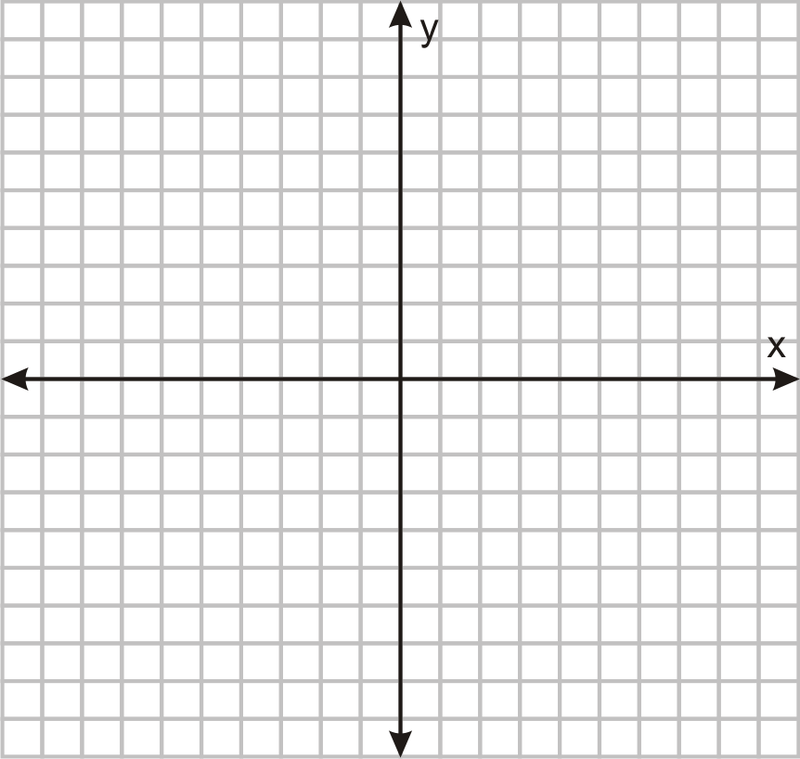
• The equations of the asymptotes are

3) Solve for the coordinates of the foci using the equation .

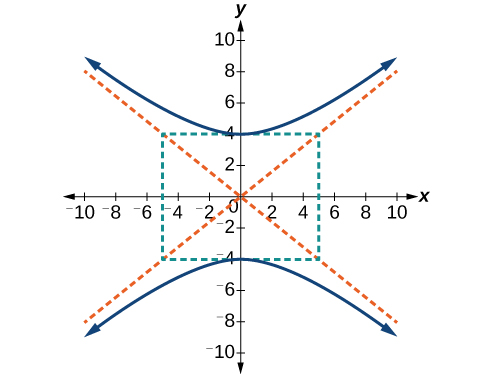
4) Plot the center, vertices, co-vertices, foci, and asymptotes in the coordinate plane, and draw a smooth curve to form the hyperbola.

Examples

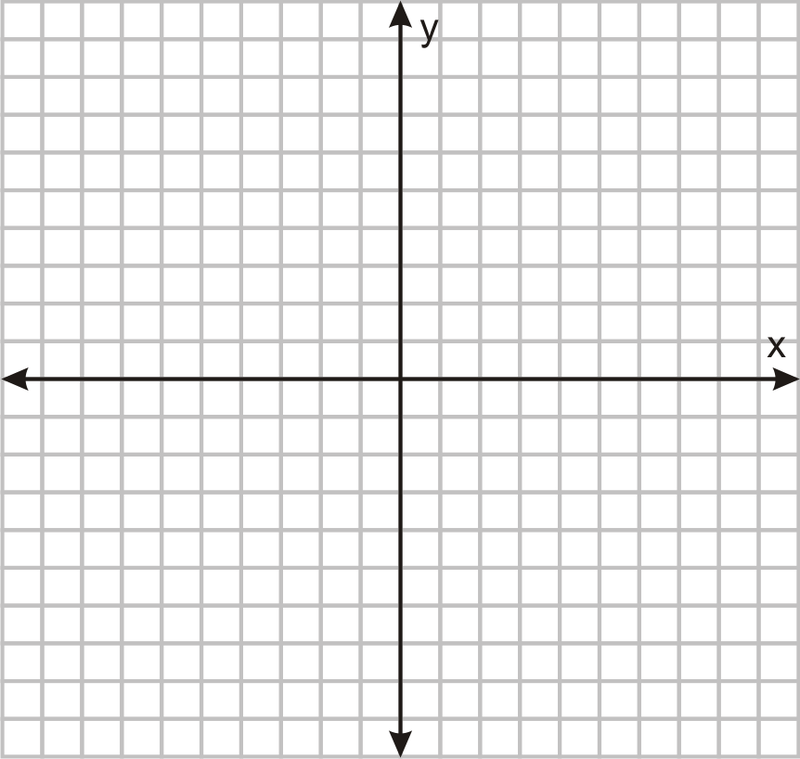
1. Graph the hyperbola given by the equation . Identify and label the vertices, co-vertices, foci, and asymptotes.



1. Given the graph of the hyperbola, find its equation.



1. Graph the hyperbola given by the equation . Identify and label the center, vertices, co-vertices, foci, and asymptotes.



1. Given the graph of the hyperbola, find its equation.

